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Inverse Exponential Power distribution: Theory and Applications

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Abstract

Using the inverse transformation technique, we have generated a novel two-parameter inverse exponential power distribution in this paper. The skewness measure and kurtosis, cumulative distribution function, quantile function, reliability function, probability density function, and hazard rate function are only a few of the mathematical and statistical properties of the suggested distribution are discussed. LSE, CVME, and MLE methods are applied to evaluate the parameters of the new model and create asymptotic confidence intervals. The variance-covariance matrix for MLEs is obtained analytically by deriving the Fisher information matrix. Using R software, all calculations are completed. Utilizing basic graphical techniques and statistical tests on a real dataset, the potentiality of the proposed distribution is demonstrated. Comparing the suggested distribution to various alternative lifetime distributions, we have empirically demonstrated that it is offered a better fit and is more flexible.

Keywords: Exponential power distribution, Kolmogorov- Smirnov test, Maximum likelihood estimation, Reliability.

1 Introduction

Lifetime distributions are frequently used in reliability and survival analysis to measure the average lifespan of components of a system and a device. In disciplines like insurance, biology, life science, engineering, medicine, etc., lifespan distributions are often employed. A wide variety of continuous probability distributions, including exponential, gamma, and Weibull, have frequently been employed to assess lifetime data in the statistical literature. Since a few years ago, most researchers have been drawn to the exponential distribution because of its flexibility for modeling lifespan data. It has been reported that this model performs admirably in a variety of applications since there are numerous closed-form solutions to survival analyses. Under the assumption of a constant failure rate, it is simple to defend, but in reality, failure rates are not always constant. As a result, the haphazard use of the exponential lifetime model seems incorrect and unrealistic. The beta exponential distribution, which is derived from the logit of a beta random variable, was presented by (Nadarajah & Kotz, 2006). The generalized exponential (GE) distribution was created by (Gupta & Kundu, 1999). It is possible to manage data with both decreasing and increasing failure rate functions using this extended family of distributions. Modifications to the existing classical probability models have led to the establishment of new classes of models in recent years (Marshall & Olkin, 2007). Recently, there have been various attempts to create novel distributions that both extend well-known distributions and offer a lot of modeling flexibility. By adding additional parameters, a number of processes could be applied to

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create a larger family from an existing model. As a result, the statistical literature has provided a variety of classes by including one or more parameters to form novel models (Rinne, 2009; Pham & Lai, 2007). Suppose a random variable be Y, then the cumulative distribution function (CDF) of exponential distribution with parameter λ is defined by

$$F_Y(y;\lambda) = 1 - e^{-\lambda y} \; ; y > 0, \lambda > 0.$$

In statistical literature, there are numerous generalizations of the exponential distribution that can be used to create lifetime models with more flexibility. Some of the well-known generalizations include the following: (Chaudhary & Kumar, 2020b) recommended the logistic-modified exponential distribution. By modifying the exponential distribution, (Smith & Bain, 1975) established the exponential power model. The generalized exponential model was created by (Gupta & Kundu, 1999). It has a hazard function with an increasing and decreasing failure rate and is more adaptable than the exponential model. The generalized exponential distribution's PDF is

$$f_{GE}\,(x;\,\alpha,\;\lambda) = \alpha\,\lambda\,e^{-\lambda\,x}\left\{1 - e^{-\lambda\,x}\right\}^{\alpha-1}\;;\;(\alpha,\;\lambda) > 0,\;x > 0.$$

(Lan & Leemis, 2008) suggested the logistic-exponential distribution. For different values of the parameters, it has failure rates that are increasing, or decreasing, bathtub (BT)-shaped, and upside-down bathtub (UBT)-shaped. (Nadarajah & Haghighi, 2011) introduced a further addition to the exponential distribution and referred to it as a generalization of the exponential distribution. The hazard rate displays increasing and decreasing shapes, while its density can have decreasing and unimodal shapes. The exponential distribution has been further expanded by the creation of new extended exponential (EEN) distributions with monotonically rising and constant hazard rate forms (Joshi, 2015). If X be the continuous random variable, then the CDF of EEN distribution is given by

$$F(x) = 1 - \exp(-\alpha x e^{-\lambda/x}) \; ; x > 0, (\alpha, \lambda) > 0$$

The half-logistic exponential extension distribution was developed by (Chaudhary & Kumar, 2020a) using the exponential extension distribution as a basis distribution. By adopting exponential extension as the baseline model, (Joshi & Kumar, 2020b) recommended the novel exponential extension Poisson model. The truncated Cauchy power exponential distribution, an extension of the exponential distribution, was developed by (Chaudhary, Sapkota, & Kumar, 2020a). (Joshi & Kumar, 2020a) introduced the Lindley exponential power distribution, also (Joshi, Sapkota, & Kumar, 2020) suggested the logistic-exponential power distribution by utilizing the exponential power distribution as the parent model. (Chaudhary, Sapkota, & Kumar, 2020b) created the truncated Cauchy power inverse exponential distribution. As a parent distribution, (Chaudhary & Kumar, 2021) created the Arctan exponential extension distribution using the exponential extension model. (Chaudhary & Kumar, 2022) have also created half Cauchy modified exponential distribution using half Cauchy family of distribution as a baseline distribution.

In this paper, we propose a novel distribution based on the exponential power distribution that was previously proposed by (Srivastava & Kumar, 2011). This distribution is employed to analyze the software reliability data having the shape of bathtub-shaped, increasing, decreasing, and j-shaped failure rate functions for different values of the parameters. Following are the CDF and PDF for the exponential power distribution:

$$G(x) = 1 - \exp\left[1 - e^{\beta x^{\alpha}}\right] \; ; \alpha, \beta > 0, x > 0, \tag{1}$$

and

$$g(x) = \alpha\beta x^{\alpha-1} e^{\beta x^{\alpha}} \exp\left[1 - e^{\beta x^{\alpha}}\right] ; \alpha, \beta > 0, x > 0. \tag{2}$$

The many sections of this study are organized as follows: The new distribution known as the inverse exponential power (IEP) is presented in section 2 along with its mathematical and statistical properties. In Section 3, we go into great detail about the least-squares (LSE), Cramer-Von-Mises (CVME), and maximum likelihood (MLE) estimation techniques. Using a real dataset, we provide the model parameter estimated values in Section 4, together with their fisher information matrix and asymptotic confidence intervals. Furthermore, we have included examples of the various test criteria used to evaluate the suggested model's goodness of fit. Section 5 offers a few concluding remarks.

2 The Inverse Exponential Power (IEP) Distribution

In this section, we have introduced a novel model using the inverse transformation technique by inverting Equations (1) and (2) respectively. If X is a non-negative random variable that follows the IEP distribution, then the following are its CDF and PDF functions:

2.1 CDF of IEP distribution

The distribution function of inverse exponential power distribution

$$F(x;\,\alpha,\;\lambda) = \exp\left\{1 - \exp\left(\frac{\lambda}{x}\right)^{\alpha}\right\} \quad ;\;\; \alpha > 0,\;\lambda > 0, x > 0, \tag{3}$$

where $\lambda > 0$ and $\alpha > 0$ are the scale and shape parameters respectively.

2.2 PDF of IEP distribution

The PDF of the IEP distribution is

$$f(x;\,\alpha,\;\lambda) = \frac{\alpha}{\lambda}\;\left(\frac{\lambda}{x}\right)^{\alpha+1} \exp\left(\frac{\lambda}{x}\right)^{\alpha} \exp\left\{1 - \exp\left(\frac{\lambda}{x}\right)^{\alpha}\right\} \;\alpha > 0,\;\lambda > 0, x > 0. \tag{4}$$

2.3 Survival function

The survival function of IEP distribution is

$$R(x;\,\alpha,\;\lambda) = 1 - \exp\left\{1 - \exp\left(\frac{\lambda}{x}\right)^{\alpha}\right\} \quad ;\; \alpha > 0,\;\lambda > 0, x > 0. \tag{5}$$

2.4 Hazard function

$$h(x;\,\alpha,\;\lambda) = \frac{\alpha/\lambda\;\;(\lambda/x)^{\alpha+1} \exp\left(\lambda/x\right)^{\alpha} \exp\left\{1 - \exp\left(\lambda/x\right)^{\alpha}\right\}}{1 - \exp\left\{1 - \exp\left(\lambda/x\right)^{\alpha}\right\}} \quad ;\;\; \alpha > 0,\;\lambda > 0, x > 0. \tag{6}$$

Similarly, the cumulative hazard function can be calculated as

$$H\left(x\right) = -\log\left(1 - \exp\left\{1 - \exp\left(\frac{\lambda}{x}\right)^{\alpha}\right\}\right) \;;\; \alpha > 0,\;\lambda > 0, x > 0.$$

2.5 Quantile function

The quantile function of IEP is given by

$$x_p = \lambda\left(\log\left\{1 - \log(p)\right\}\right)^{-(1/\alpha)} \quad ;\; 0 < p < 1. \tag{7}$$

Random deviate generation is given by

$$x = \lambda\;\left[\log\left\{1 - \log(u)\right\}\right]^{-(1/\alpha)} \quad ;\; 0 < u < 1.$$

where u has the uniform $U(0,1)$ distribution.

2.6 Median of IEP distribution

Simply substituting $p = 0.5$ in Equation (7) yields the median of X from the IEP distribution as $Median = \lambda\left(0.1142\right)^{-(1/\alpha)}$.

2.7 Skewness and Kurtosis

(Kennedy & Keeping, 1962) developed the quartile-based Bowley's measure of skewness as follows:

$$S_k\,(B) = \frac{Q\,(1/4) + Q\,(3/4) - 2Q\,(1/2)}{Q\,(3/4) - Q\,(1/4)},$$

and the coefficient of kurtosis calculated by (Moors, 1988) using octiles is as follows:

$$K_u\,(M) = \frac{Q\,(0.875) + Q\,(0.375) - Q\,(0.125) - Q\,(0.625)}{Q\,(3/4) - Q\,(1/4)}.$$

Figure 1 displays the PDF and hazard function graphs for the IEP distribution for various parameter values. The density function of the IEP distribution can take on several shapes, as seen in Figure 1 (left panel), depending on the numerous parameter values. The hazard rate is depicted in Figure 1 (right panel) as an increasing, inverted bathtub, and decreasing shape.

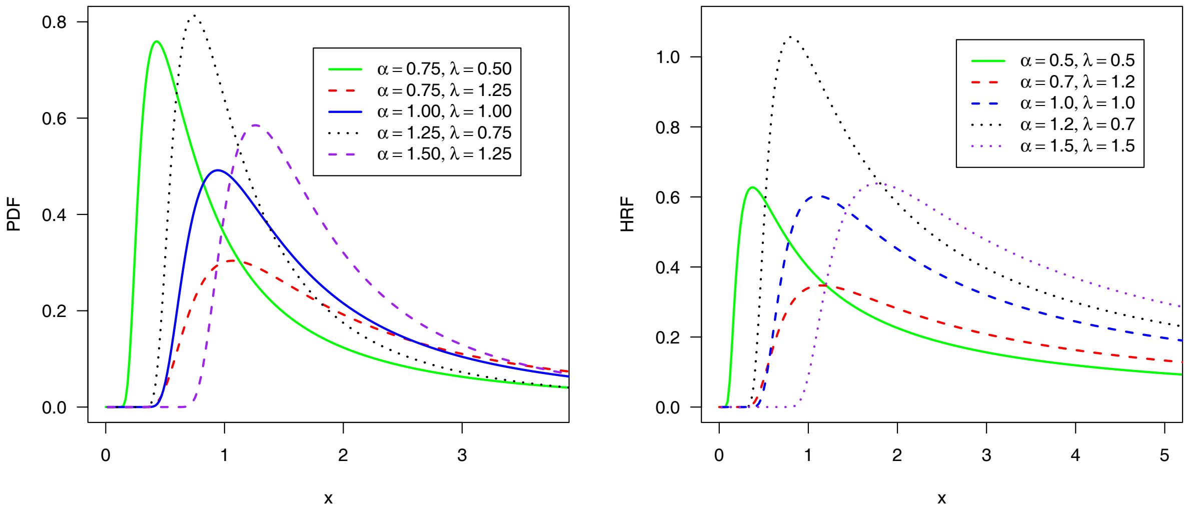

Figure 1: Hazard function (right panel) and PDF (left panel) graphs for numerous parameter values.

3 Parameter Estimation

3.1 Method of Maximum Likelihood Estimation (MLE)

Let $X_1, ..., X_n$ represent a random sample of size n independently generated, uniformly distributed taken from the IEP model with parameters α and λ but unknown. Using the PDF in Equation (4), the likelihood function of the IEP is as follows:

$$\ell\,(\alpha, \lambda|\underline{x}) = \; n \log \alpha \; + \; n\alpha \log \lambda - (\alpha+1) \sum_{i=1}^{n} \log x_i \; + \lambda^{\alpha} \sum_{i=1}^{n} x_i^{-\alpha} + n - \sum_{i=1}^{n} \exp\left\{(\lambda/\,x_i)^{\alpha}\right\} \tag{8}$$

By differentiating Equation (8) and equating to zero, we get the solutions for the following nonlinear equations to estimate the unknown parameters of the $IEP\,(\alpha, \lambda)$.

$$\frac{\partial \ell}{\partial \alpha} \; = \; \frac{n}{\alpha} \; + \; n \log \lambda - \sum_{i=1}^{n} \log x_i \; + \sum_{i=1}^{n} (\lambda/\,x_i)^{\alpha} \; \log(\lambda/x_i)\left\{1 - \exp\,(\lambda/x_i)^{\alpha}\right\} = 0 \tag{9}$$

$$\frac{\partial \ell}{\partial \lambda} \; = \; \frac{n\;\alpha}{\lambda} \; + \sum_{i=1}^{n} \alpha\; \lambda^{\alpha-1}\; x_i^{\alpha}\; \left\{1 - \exp\,(\lambda\,/x_i)^{\alpha}\right\} = 0 \tag{10}$$

It is difficult to solve the Equations (9) and (10) for α and λ. Therefore, one can solve these equations using the Newton-Raphson iteration method or any other suitable computer tools like R, Mathematica, Matlab, or others. Let's symbolize the parameter vector by $\underline{\tau} = (\alpha, \lambda)$ and the associated MLE for $\underline{\tau}$ as $\hat{\underline{\tau}} = (\hat{\alpha}, \hat{\lambda})$, then the asymptotic normality results in,

$$(\hat{\underline{\tau}} - \hat{\tau}) \rightarrow N_2 \left[0, (I(\tau))^{-1}\right].$$

Here, $I(\tau)$ stands for Fisher's information matrix which is expressed by,

$$I(\underline{\tau}) = -\begin{bmatrix} E\left(\frac{\partial^2 l}{\partial \alpha^2}\right) & E\left(\frac{\partial^2 l}{\partial \alpha \partial \lambda}\right) \\ E\left(\frac{\partial^2 l}{\partial \lambda \partial \alpha}\right) & E\left(\frac{\partial^2 l}{\partial \lambda^2}\right) \end{bmatrix}$$

Further differentiating Equations (9) and (10) we get,

$$\frac{\partial^2 l}{\partial \alpha^2} = -\frac{n}{\alpha^2} + \sum_{i=1}^{n} (\lambda/x_i)^{\alpha} \left[\ln(\lambda/x_i)\right]^2 \left[1 - (\lambda/x_i)^{\alpha} e^{(\lambda/x_i)^{\alpha}} + e^{(\lambda/x_i)^{\alpha}}\right]$$

$$\frac{\partial^2 l}{\partial \lambda^2} = -\frac{n\alpha}{\lambda^2} + \alpha(\alpha - 1) \sum_{i=1}^{n} \frac{1}{x_i^2} (\lambda/x_i)^{\alpha-2} \left[1 - e^{(\lambda/x_i)^{\alpha}}\right] - \sum_{i=1}^{n} (\lambda/x_i)(\lambda/x_i)^{2\alpha-1} e^{(\lambda/x_i)^{\alpha}} \ln(\lambda/x_i)$$

$$\frac{\partial^2 l}{\partial \alpha \partial \lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n} (\lambda/x_i)^{\alpha} \left[\left[\{(\lambda/x_i)\ln(\lambda/x_i) + \ln(\lambda/x_i)\}\alpha + 1\right] e^{(\lambda/x_i)^{\alpha}} - \ln(\lambda/x_i)^{\alpha-1}\right]$$

$$\frac{\partial^2 l}{\partial \lambda \partial \alpha} = \frac{n}{\lambda} - \sum_{i=1}^{n} (\lambda/x_i)^{\alpha} \left[\left[\{(\lambda/x_i)\ln(\lambda/x_i) + \ln(\lambda/x_i)\}\alpha + 1\right] e^{(\lambda/x_i)^{\alpha}} - \ln(\lambda/x_i)^{\alpha-1}\right]$$

Since we don't know $\underline{\tau}$, it is of no value in practice that the MLE has asymptotic variance $(I(\underline{\tau}))^{-1}$. So, using the estimated parameter values, the asymptotic variance can be approximated. Typically, the observed Fisher information matrix $\Psi(\hat{\underline{\tau}})$ is used to estimate the information matrix $I(\underline{\tau})$ provided by

$$\Psi(\hat{\underline{\tau}}) = -\begin{bmatrix} \frac{\partial^2 l}{\partial \alpha^2} & \frac{\partial^2 l}{\partial \alpha \partial \lambda} \\ \frac{\partial^2 l}{\partial \alpha \partial \lambda} & \frac{\partial^2 l}{\partial \lambda^2} \end{bmatrix}_{|(\hat{\alpha}, \hat{\lambda})} = -H(\underline{\tau})_{|(\underline{\tau} = \hat{\tau})}$$

where H stands for the Hessian matrix. The observed information matrix is generated by the Newton-Raphson method with the goal of maximizing the likelihood and the Variance-Covariance Matrix can thus be represented through,

$$\left[-H(\underline{\tau})_{|(\underline{\tau} = \hat{\underline{\tau}})}\right]^{-1} = \begin{bmatrix} \mathrm{var}(\hat{\alpha}) & \mathrm{cov}(\hat{\alpha}, \hat{\lambda}) \\ \mathrm{cov}(\hat{\alpha}, \hat{\lambda}) & \mathrm{var}(\hat{\lambda}) \end{bmatrix}.$$

Let $Z_{\delta/2}$ stand for the upper percentile of standard normal variate. In light of the MLEs' asymptotic normality, it is possible to construct the approximate $100(1 - \delta)\%$ confidence intervals for α and λ as follows:

$$\hat{\lambda} \pm Z_{\delta/2} \sqrt{\mathrm{var}(\hat{\lambda})}$$

and

$$\hat{\alpha} \pm Z_{\delta/2} \sqrt{\mathrm{var}(\hat{\alpha})}$$

3.2 Method of Least-Square Estimation (LSE)

The weighted least square and the ordinary least square estimators are two further estimation methods recommended by (Swain, Venkatraman, & Wilson, 1988) which are applied to evaluate the Beta distributions' parameters. In this case, we estimate the IEP distribution's parameters using the same method. By minimizing (8) with respect to α and λ, it is possible to determine the LSEs of the unknown parameters α and λ of the IEP model.

$$M\left(\underline{X};\alpha,\beta,\lambda\right)=\sum_{i=1}^{n}\left[F(X_{(i)})-\frac{i}{n+1}\right]^{2}.$$

If $\underline{X}=(X_1,\ldots,X_n)$ is a random sample of size n drawn from a distribution function $F(.)$, then $F(X_i)$ indicates the distribution function for the ordered random variables $\mathrm{X}_{(1)}<\ldots<\ \mathrm{X}_{(\mathrm{n})}$. By minimizing (11) with respect to α and λ, one can get the LSEs of α and λ say $\hat{\alpha}$ and $\hat{\lambda}$ respectively.

$$M\left(\underline{X};\alpha,\lambda\right)=\sum_{i=1}^{n}\left[\exp\left\{1-\exp\left(\lambda/x_{(i)}\right)^{\alpha}\right\}\ -\frac{i}{n+1}\right]^{2}. \tag{11}$$

Differentiating Equation (11) with respect to α and λ we get,

$$\frac{\partial M}{\partial\alpha}=-2\lambda\sum_{i=1}^{n}\left[\exp\left\{1-\exp\left(\lambda/x_{(i)}\right)^{\alpha}\right\}\ -\frac{i}{n+1}\right]\left[\frac{1}{x_i}\exp\left(\lambda/x_{(i)}\right)^{\alpha}\exp\left\{1-\exp\left(\lambda/x_{(i)}\right)^{\alpha}\right\}\right]$$

$$\frac{\partial M}{\partial\lambda}=-2\alpha\sum_{i=1}^{n}\left[\exp\left\{1-\exp\left(\lambda/x_{(i)}\right)^{\alpha}\right\}\ -\frac{i}{n+1}\right]\left[\frac{1}{x_i}\exp\left(\lambda/x_{(i)}\right)^{\alpha}\exp\left\{1-\exp\left(\lambda/x_{(i)}\right)^{\alpha}\right\}\right]$$

By setting the above two equations to zero and solving them we will get LSEs.

3.3 Cramer-Von-Mises estimation (CVME) Method

By minimizing the function (12) with respect to the unknown parameters α and λ, the CVMEs of α and λ are produced.

$$\begin{aligned}D\left(\underline{X};\alpha,\lambda\right)&=\frac{1}{12n}+\sum_{i=1}^{n}\left[F\left(x_{(i)}|\alpha,\lambda\right)-\frac{2i-1}{2n}\right]^{2}\\&=\frac{1}{12n}+\sum_{i=1}^{n}\left[\exp\left\{1-\exp\left(\lambda/x_{(i)}\right)^{\alpha}\right\}-\frac{2i-1}{2n}\right]^{2}.\end{aligned} \tag{12}$$

Differentiating equation (12) with respect to α and λ we get,

$$\frac{\partial D}{\partial\alpha}=-2\lambda\sum_{i=1}^{n}\left[\exp\left\{1-\exp\left(\lambda/x_{(i)}\right)^{\alpha}\right\}\ -\frac{2i-1}{2n}\right]\left[\frac{1}{x_i}\exp\left(\lambda/x_{(i)}\right)^{\alpha}\exp\left\{1-\exp\left(\lambda/x_{(i)}\right)^{\alpha}\right\}\right]$$

$$\frac{\partial D}{\partial\lambda}=-2\alpha\sum_{i=1}^{n}\left[\exp\left\{1-\exp\left(\lambda/x_{(i)}\right)^{\alpha}\right\}\ -\frac{2i-1}{2n}\right]\left[1/x_{(i)}\exp\left(\lambda/x_{(i)}\right)^{\alpha}\exp\left\{1-\exp\left(\lambda/x_{(i)}\right)^{\alpha}\right\}\right]$$

The CVM estimators can be obtained by concurrently solving

$$\frac{\partial D}{\partial\alpha}=0\ and\ \frac{\partial D}{\partial\lambda}=0.$$

4 Application with a real dataset

A real data set of the relief time of 20 patients taking an analgesic is taken for the application of the suggested model and this data set was reported by (Gross & Clark, 1975). Data are as follows:

1.4, 1.1, 1.7, 1.3, 1.8, 1.9, 2.2, 1.6, 2.7, 1.7, 1.8, 4.1, 1.2, 1.5, 3, 1.4, 2.3, 1.7, 2.0, 1.6

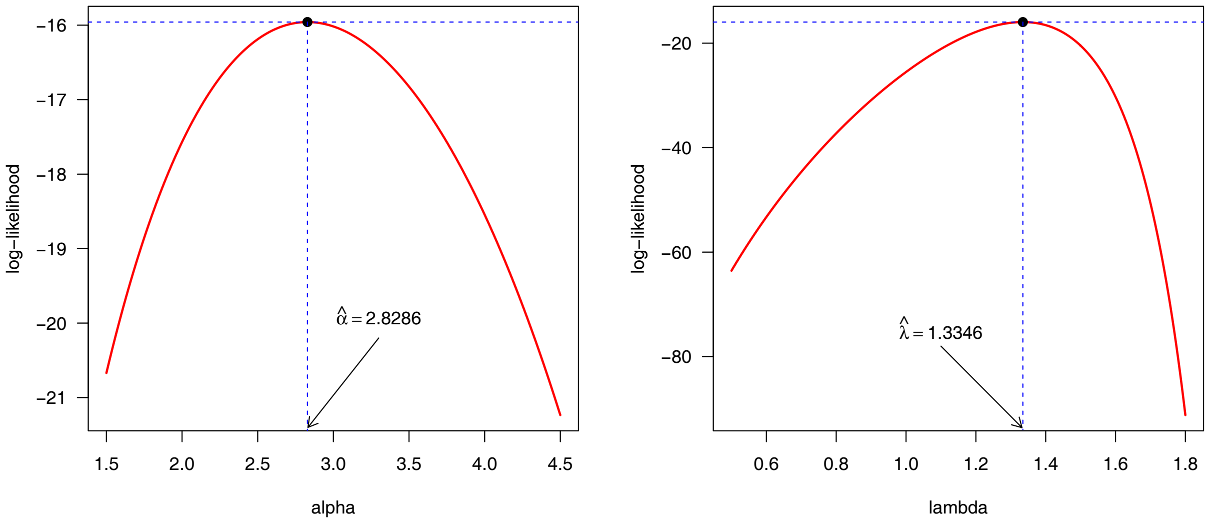

Figure 2: Plots of profile for the parameters α and λ of IEP distribution.

Table 1: 95% confidence interval, MLE, and SE

Parameter	**MLE**	**SE**	**95% ACI**
alpha	2.82863	0.51181	(1.8255 3.8318)
lambda	1.33455	0.06552	(1.2061 1.4629)

Figure 2 shows the profile log-likelihood function plots for the parameters α and λ and it can be seen that the ML estimates can be derived individually. Also, we have presented the contour plot for α and λ in Figure 4 (right panel). By using R software's **optim()** function(R Core Team, 2022; Lambert, 2018) and maximizing the likelihood function (8), the maximum likelihood estimates are computed directly. Table 1 displays the 95% asymptotic confidence intervals, the MLEs for α and λ, together with associated standard errors (SE). Hence the Hessian variance-covariance matrix is obtained as,

$$\left[-H\left(\underline{\tau}\right)\big|_{(\underline{\tau}=\hat{\underline{\tau}})}\right]^{-1} = \begin{bmatrix} 0.26195 & -0.00869 \\ -0.00869 & 0.00429 \end{bmatrix}$$

Table 1 displays the estimated values of the IEP distribution's parameters for the under-study data using the CVE, LSE, and MLE methods, as well as the associated AIC, KS, and negative log-likelihood criteria. The Q-Q plot, the fitted distributions' density function, and the histogram for the CVM, LSE, and MLE estimate methods are shown in Figure 3. Figure 4 shows the Q-Q plot of the IEP distribution. The distribution has been observed to fit the data exactly.

Table 2: Log-likelihood, AIC, BIC, KS, and estimated parameters

Estimation Method	**alpha**	**lambda**	**LL**	**AIC**	**BIC**	**KS(p-value)**
MLE	2.8286	1.3346	-15.958	35.9165	37.9079	0.1406(0.8243)
LSE	2.9662	1.3745	-16.292	36.5832	38.5747	0.1101(0.9685)
CVME	3.2314	1.398	-17.427	38.8543	40.8458	0.1030(0.9838)

We have fitted the IEP distribution and some selected distributions for the comparison, which are Generalized Gompertz (GGZ) distribution (El-Gohary, Alshamrani, & Al-Otaibi, 2013), Generalized Exponential Extension (GEE) distribution (Lemonte, 2013), Generalized Exponential (GE) distribution (Gupta & Kundu, 1999) and Generalized Rayleigh distribution (Kundu & Raqab, 2005).

In Table 3, the value of the AIC, BIC, CAIC, and HQIC as well as the negative log-likelihood value are displayed. We get to the conclusion that compared to other models, the suggested model offers a superior fit to the real data set. By displaying the fitted density functions and the histogram,

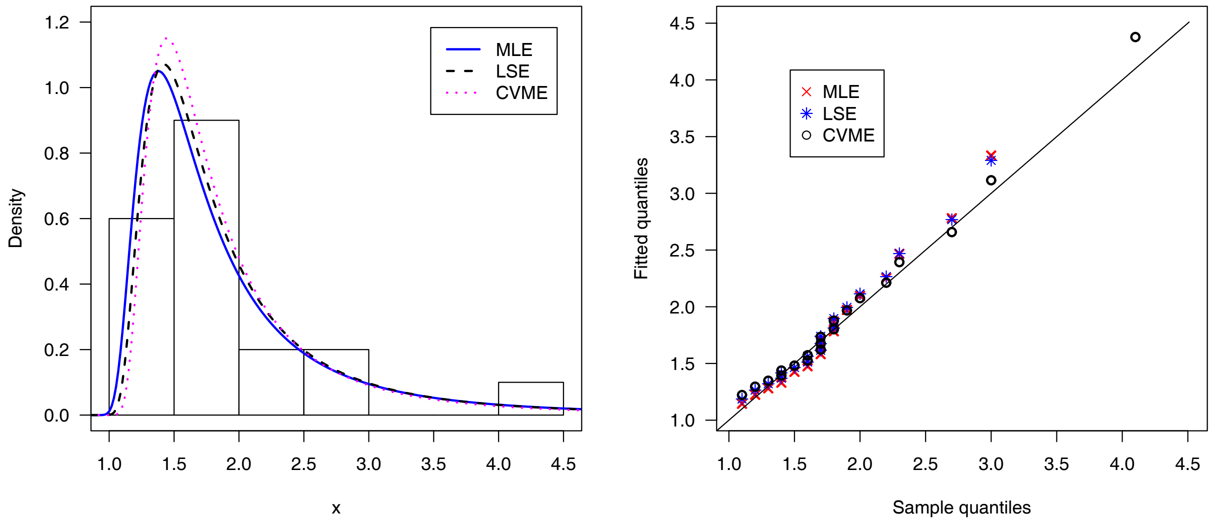

Figure 3: The density function and the Histogram of fitted distributions for MLE, LSE and CVM (left panel) and sample quantiles and fitted quantiles (right panel) of IEP distribution.

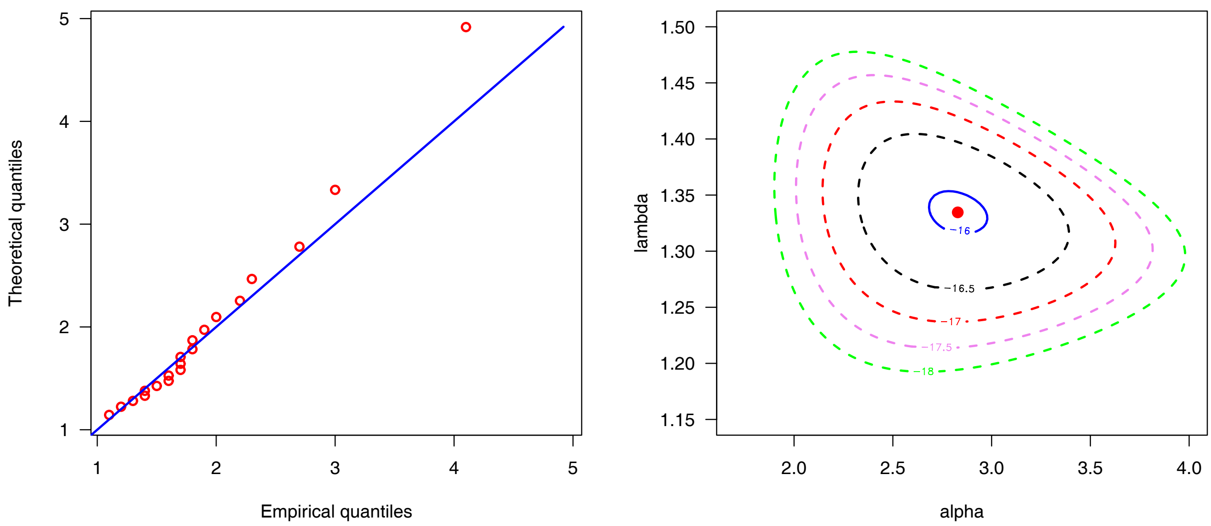

Figure 4: Contour plot (right panel) and the Q-Q plot (left panel) for IEP distribution.

Table 3: Log-likelihood, AIC, BIC, CAIC and HQIC statistics

Distribution	**LL**	**AIC**	**BIC**	**CAIC**	**HQIC**
IEP	-15.958	35.9165	37.9079	36.5481	36.3052
GEE	-16.11	38.2206	41.2078	39.7206	38.8037
GE	-16.261	36.5212	38.5127	37.2271	36.91
GGZ	-16.39	38.7805	41.7677	40.2805	39.3636
GR	-18.402	40.8045	42.796	41.5104	41.1933

Figure 5 contrasts the empirical distribution function that produces the same or better results with the distribution function for the various models. The provided data sets thus show that, compared to other alternatives, the proposed distribution provides a better fit and more trustworthy findings. The test statistics values for the various models are shown in Table 4 together with the relevant p-

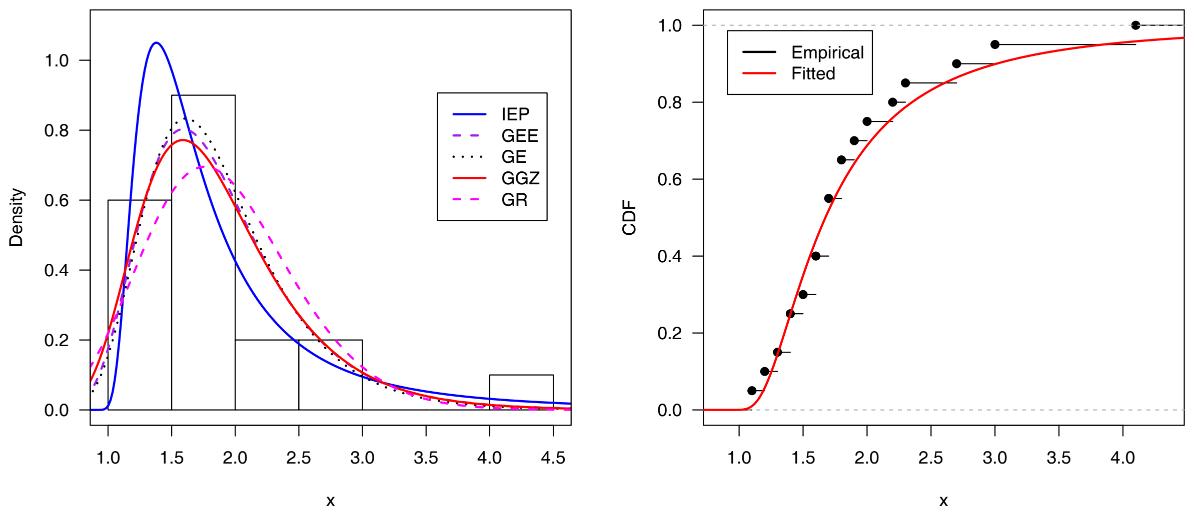

Figure 5: Empirical CDF with estimated CDF (right panel) and the PDF and the Histogram of fitted distributions (left panel).

values for the Kolmogorov-Simnorov (KS), Anderson-Darling (AD), and Cramer-Von Mises (CVM) statistics. The result confirms that the suggested model has the smallest test statistic value and a larger p-value, leading us to believe that the inverse exponential power distribution is preferable in terms of goodness-of-fit.

Table 4: The p-value and related statistics for goodness of fit

Distribution	KS(p-value)	AD(p-value)	CVM(p-value)
IEP	0.1406(0.8243)	0.0519(0.8699)	0.2914(0.9439)
GEE	0.1363(0.8516)	0.0501(0.8812)	0.2903(0.9447)
GE	0.1343(0.8633)	0.0477(0.8954)	0.3105(0.9293)
GGZ	0.1305(0.8852)	0.0492(0.8866)	0.3111(0.9288)
GR	0.1900(0.4655)	0.1272(0.4707)	0.7343(0.5290)

5 Conclusion

The Inverse Exponential Power (IEP) distribution using the inverse transformation technique is a new extension of the exponential power model that we have introduced in this study. We've discussed about a few of the IEP model's statistical and mathematical properties. The suggested model is flexible and has growing, decreasing, and upside-down bathtub hazard functions, according to the graphical analysis of PDF and HRF. CVME, LSE, and MLE methods are used to evaluate the parameters of the new distribution and create asymptotic confidence intervals. We have also used real data set to demonstrate the use of the IEP distribution, and we have discovered that it performs better in terms of fitting than a few chosen models. For practitioners in theory and applied statistics, it can be an alternative model.

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